APPENDIX 5

The case of the $N = 4n + 1$ order

Preliminary observation - To preserve a lattice with n + 1 cells with coefficients a_i, with $0 \leq i \leq n$, as defined for the 4n + 3 order, we present the case of the $4n + 5$ order.

Let L_i, $i = 0, ..., 3$ be the polyphase components of a null ISI $4n + 3$ order $L(z)$ filter. It is possible to associate, with this filter, a lattice built as in Figure 8a of the patent with a set of coefficients α_i, $i = 0, ..., n$. We have $L_2(z) = \hat{L}_1(z)$ and $L_3(z) = \hat{L}_0(z)$.

The filter $F(z)$ whose components F_i $i=0,\ldots,3$ are defined by

$$F_0(z) = L_0(z)\,, \tag{56}$$

$$F_1(z) = z^{-1}\hat{L}_0(z)\,, \tag{57}$$

$$F_2(z) = L_1(z)\,, \tag{58}$$

$$F_3(z) = \hat{L}_1(z)\,, \tag{59}$$

is a null ISI, $4n + 5$ order linear phase filter. Furthermore, the Theorem 2 in Appendix 2 of the patent establishes the fact that such filters are necessarily obtained in this way. Of course, the synthesis of such a filter is done so as to optimize the coefficients α_i, $i = 0, ..., n$ to obtain the right frequency specifications for the filter $F(z)$, not for the filter $L(z)$! This therefore leads to the obtaining of diagrams for sending and reception associated with $F(z)$ from the lattice built with the coefficients α_i and the associated inverted lattice.

The type I polyphase breakdown applied to this filter is expressed by the following expression:

$$F(z) = L_0(z^4) + z^{-1}[z^{-4}\hat{L}_0(z^4)] + z^{-2}L_1(z^4) + z^{-3}\hat{L}_1(z^4) \tag{60}$$

It can be ascertained that, for the sending part, this equation may be expressed by the lattice diagram of Figure 20 wherein a delay is applied. This delay is given by z^{-3} at output of $F(z)$.

For the reception part as in the case of the *4n* + 3 order, it is possible to make a 4-lattice system that ensures the totality of the properties sought.

Similarly, simplifications appear if we consider the system known as the back-to-back system. The input signal of the reception filter can then be written as follows:

$$Y(z) = [z^{-3}L_0(z^4) + z^{-8}[z^{-4}\hat{L}_0(z^4)] + z^{-5}L_1(z^4) + z^{-6}\hat{L}_1(z^4)]X(z^4) \quad (61)$$

To recover the input signal, apart from a delay, we then implement the device of Figure 21.

Referring to Figure 21, the signals after this decimation are, from top to bottom, given by:

$$\begin{array}{ll} Y(z)\downarrow_4 = z^{-2}\hat{L}_0(z)X(z), & z^{-1}Y(z)\downarrow_4 = z^{-1}L_0(z)X(z), \\ z^{-2}Y(z)\downarrow_4 = z^{-2}\hat{L}_1(z)X(z), & z^{-3}Y(z)\downarrow_4 = z^{-2}L_1(z)X(z) \end{array} \quad (62)$$

From top to bottom, the signals at input of the lattice are respectively given by:

$$z^{-2}[\hat{L}_0(z) + \hat{L}_1(z)]X(z) \text{ et } z^{-2}[L_0(z) - L_1(z)]X(z).$$

Thus, as in the case of the *4n* + 3 order but this time with a processing delay of n + 2 samples, we have an output signal which, for $g = 1/(2\gamma)$ is identical to that of the input.

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